Problem of the Week — December 8, 2015

This is the last one, problem solvers. The student who submits the first correct and thoroughly-written solution will receive a $15 gift card to the Kenyon bookstore. More importantly, you will have earned undeniable bragging rights.

PROBLEM: Suppose f is a real-valued continuously differentiable function on (a, b) with $0 < a < b$. Furthermore, suppose $f(a) = f(b) = 0$. If we let $I = \int_a^b [f(x)]^2 dx$, then show that

$$\int_a^b [f'(x)]^2 dx \ \geq \ \frac{I}{4b^2} \ .$$

Hint: An intermediate step might be to find the value of $\int_a^b x f(x) f'(x) dx$.

Solutions accepted until 11:59 pm 12/18/15

You must submit complete solutions to Brian Jones (Hayes 303) either via email or hard copy; however, if you submit a hard copy, it must have a time-stamp (i.e. either electronic proof of time printed or a faculty signature verifying the time submitted.) Please, if you write out a solution by hand and scan, write neatly and in black ink so your work is easily readable.