1. Solve for $y$: $2y^2 - 3y - 2 = 0$

Solution: $2y^2 - 3y - 2 = (2y + 1)(y - 2)$, so for $2y^2 - 3y - 2$ to equal 0, it must be the case that $2y + 1 = 0$ or $y - 2 = 0$. In the former case, $y = -1/2$ and in the latter case, $y = 2$.

2. Solve for $x$: $\sqrt{6x + 7} = x + 2$

Solution: We’ll begin by squaring both sides of the equation: $6x + 7 = (x + 2)^2$. Now let’s expand the right side: $6x + 7 = x^2 + 4x + 4$. Moving all terms to the right side, we obtain $0 = x^2 - 2x - 3$, which we can again solve by factoring. $x^2 - 2x - 3 = (x - 3)(x + 1)$. Following the same reasoning as in the first problem, we see that $x = 3$ or $x = -1$.

(Note that a quick check of both solutions by plugging in the respective numbers shows that both are valid. Sometimes this is not the case when one squares a square root, so it’s something to watch out for!)

3. Solve for $x$: $-5 < \frac{1}{2}(3x + 1) \leq 7$

Solution: We begin by multiplying both inequalities through by 2. This is allowed and does not change the sign since $2 > 0$. $-10 < 3x + 1 \leq 14$. We proceed by subtracting 1 from all three terms. (This is always allowed.) $-11 < 3x \leq 13$. Finally, dividing by 3, we see that $\frac{-11}{3} < x \leq \frac{13}{3}$.

4. Solve for $x$: $|2x + 3| < 13$

Solution: This is an abbreviation for $-13 < 2x + 3 < 13$. Now we can proceed as in the previous problem: $-16 < 2x < 10$, so $-8 < x < 5$.

5. Which of the following are graphs of functions?

Solution: The graphs in Figures I and III are functions, since they pass the vertical line test. (In figures II and IV, it is possible to draw a vertical line which intersects the graph twice.)
6. Match each equation with its graph:

(a) \( y = -2x^2 + 3x + 1 \)

Solution: Figure II (It is the only parabolic-looking graph.)

(b) \( y = \frac{1}{2}e^{-x} - 1 \)

Solution: Figure III (As \( x \) gets large, \( e^{-x} \) is near 0, so the graph should approach \( y = -1 \).)

(c) \( y = \log_5 x \)

Solution: Figure IV (Note particularly that \( \log_5 1 = 0 \).)

(d) \( y = 1 - \sin x \)

Solution: Figure I (This one should have been pretty obvious.)

7. Let \( f(x) = 5x^2 + 4x \). Evaluate each of the following:

(a) \( f(0) \)

Solution: \( f(0) = 5 \cdot 0 + 4 \cdot 0 = 0 \)

(b) \( f(3) \)

Solution: \( f(3) = 5 \cdot 9 + 4 \cdot 3 = 45 + 12 = 57 \)

(c) \( f(-1) \)

Solution \( f(-1) = 5 \cdot 1 + 4 \cdot -1 = 5 - 4 = 1 \)

(d) \( f(t) \)

Solution: \( f(t) = 5t^2 + 4t \)

(e) \( f(t-1) \)

Solution: \( f(t-1) = 5(t-1)^2 + 4(t-1) = (5t^2 - 10t + 5) + 4t - 4 = 5t^2 - 6t + 1 \)

(f) \( \frac{f(a+h) - f(a)}{h} \)

Solution \( f(a+h) = 5(a+h)^2 + 4(a+h) = 5a^2 + 10ah + 5h^2 + 4a + 4h. \) And of course, \( f(a) = 5a^2 + 4a, \) so \( f(a+h) - f(a) = (5a^2 + 10ah + 5h^2 + 4a + 4h) - (5a^2 + 4a) = 10ah + 5h^2 + 4h. \) And finally, \( \frac{f(a+h) - f(a)}{h} = \frac{10ah + 5h^2 + 4h}{h} = 10a + 5h + 4. \)

8. Find the \( x \)- and \( y \)-intercepts of \( y = (x - 2)^2(x + 2)(x + 4). \)

Solution: To find the \( x \)-intercepts, we set \( y = 0. \) \( 0 = (x - 2)^2(x + 2)(x + 4). \)

For that product to be 0, we need at least one of the factors to be 0, so \( x - 2 = 0, \) \( x + 2 = 0, \) or \( x + 4 = 0. \) Thus, the \( x \)-intercepts of the graph are \( x = 2, \) \( x = -2, \) and \( x = -4. \)

Finding the \( y \)-intercepts is easier. We set \( x = 0 \) and see that \( y = (-2)^2(2)(4) = 32. \)
9. Factor $x^3 + 4x^2 - 12x$ as completely as possible.

Solution: $x^3 + 4x^2 - 12x = x(x^2 + 4x - 12) = x(x + 6)(x - 2)$.

10. Let $A$ be the point $(-2, 1)$, let $B$ be the point $(2, 3)$, and let $C$ be the point $(3, 1)$. First plot these points and draw the triangle $ABC$. Then use the distance formula to find the three sides of the right triangle $ABC$. Finally, verify the Pythagorean Theorem for this triangle.

Solution: I’ll leave the diagramming out, but the calculations are as follows:

$|AB| = \sqrt{(2 - (-2))^2 + (3 - 1)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$

$|AC| = \sqrt{(3 - (-2))^2 + (1 - 1)^2} = \sqrt{25} = 5$

$|BC| = \sqrt{(3 - 2)^2 + (1 - 3)^2} = \sqrt{1 + 4} = \sqrt{5}$

Finally, we check the Pythagorean Theorem: $(2\sqrt{5})^2 + (\sqrt{5})^2 = 20 + 5 = 25 = 5^2$.

11. Find the vertical and horizontal asymptotes of $y = \frac{3x + 5}{x - 6}$.

Solution: Some basic knowledge of asymptotes tells us that $y = \frac{3x + 5}{x - 6}$ will have a vertical asymptote when the denominator is 0, so $x = 6$ is a vertical asymptote of this function. As for the horizontal asymptote, you can either use algebra ($\frac{3x + 5}{x - 6} = \frac{3x - 18 + 23}{x - 6} = \frac{3x - 18}{x - 6} + \frac{23}{x - 6} = 3 + \frac{23}{x - 6}$) or just some common sense (“As $x$ gets very large, the $+5$ and $-6$ will be negligible compared to the size of $x$.”) to see that as $x$ gets very large (positive or negative), $y$ will be close to 3, so $y = 3$ is the horizontal asymptote of the graph.

12. Solve for $x$: $e^{2x} = 8$

Solution: We begin by taking the natural logarithm of both sides. $\ln e^{2x} = \ln 8$.

But of course, $\ln(e^{\text{whatever}}) = \text{whatever}$, so $2x = \ln 8$ and $x = (\ln 8)/2$.

13. Solve for $x$: $2^{3x-4} = 5$

Solution: We proceed in the same way as above, but instead of using the natural logarithm (base $e$), we use the logarithm base 2. $\log_2(2^{3x-4}) = \log_2 5$, so as before $3x - 4 = \log_2 5$. At this point, it is very easy to see that $x = (4 + \log_2 5)/3$.

14. Solve for $x$: $\log_2(3x - 4) = 5$

Solution: Just as $\log_2(2^{\text{whatever}}) = \text{whatever}$, it’s also the case that $2^{\log_2(\text{whatever})} = \text{whatever}$, so we can do something similar to what we did above. $2^{\log_2(3x-4)} = 2^5$, so $3x - 4 = 32$, and so $3x = 36$ and $x = 12$. 

Page 3
15. Solve for \( x \): \( \ln x + \ln(x + 3) = 1 \)

Solution: It’s a property of all logarithmic functions that \( \log A + \log B = \log AB \) and we can use that here: \( \ln x + \ln(x + 3) = \ln(x(x + 3)) = 1 \). Now we can do what we did the preceding problem: \( e^{\ln(x(x+3))} = e^1 \), so \( x(x+3) = e \).

Finally, we solve this like any quadratic problem. (Remember: \( e \) is just a number.) \( x^2 + 3x - e = 0 \), so we can use the quadratic formula: \( x = \frac{-3 \pm \sqrt{9 - (4(-e))}}{2} = \frac{-3 \pm \sqrt{9 + 4e}}{2} \). The last thing to note here is that we can’t take the logarithm of a negative number (remember the original statement of the problem!), so the answer is \( x = \frac{-3 + \sqrt{9 + 4e}}{2} \).

Whew!

16. Convert the radian angle measures \( \frac{5\pi}{4}, \frac{5\pi}{6}, \) and \( \frac{-5\pi}{2} \) to degree measures.

Solution: \( 225^\circ, 150^\circ, \) and \( -450^\circ \).

17. Find the exact value of \( \tan(\frac{4\pi}{3}) + \cos(\frac{4\pi}{3}) \) without using a calculator.

Solution: \( \sqrt{3} - 1/2 \). How did we get there? Well, \( \frac{4\pi}{3} \) is \( 240^\circ \), so if we draw a unit circle and a ray at \( 240^\circ \), we see that we get a 30-60-90 triangle, with the long leg in the vertical and the short leg in the horizontal. Furthermore, both \( x \) and \( y \) are negative. That tells us that \( \sin(\frac{4\pi}{3}) = -\sqrt{3}/2 \) and \( \cos(\frac{4\pi}{3}) = -1/2 \). Since \( \tan \theta = \sin \theta / \cos \theta \), \( \tan(\frac{4\pi}{3}) = \sqrt{3} \).

18. Below are the graphs of the six basic trigonometric functions. Which graph represents which function?

Solution: I – sec \( x \); II – tan \( x \); III – cos \( x \); IV – csc \( x \); V – cot \( x \); VI – sin \( x \).

19. Let \( \theta \) be the angle between the \( x \)-axis and a segment joining the origin to \( (-3, 4) \). Find \( \sin \theta \).

Solution: Since the length of the segment is 5 (remember 3-4-5 triangles?), it intersects the unit circle at \( (-3/5, 4/5) \). Since the value of the sine is the value of the \( y \)-coordinate in unit-circle trigonometry, \( \sin \theta = 4/5 \).

20. Verify that \( \csc \theta - \cot \theta = \frac{\sin \theta}{1 + \cos \theta} \). (Hint: Write everything in terms of \( \sin x \) and \( \cos x \) first.)

Taking the hint, we start by writing: \( \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} = \frac{\sin \theta}{\sin \theta} = \frac{1 - \cos \theta}{\sin \theta} \). Now we can cross-multiply to get rid of the fractions: \( (1 - \cos \theta)(1 + \cos \theta) = (\sin \theta)^2 \). Now if we multiply out the left side, we get \( 1 - \cos \theta + \cos \theta - (\cos \theta)^2 = 1 - (\cos \theta)^2 = (\sin \theta)^2 \), which is a variant on the basic trigonometric identity: \( 1 = (\sin \theta)^2 + (\cos \theta)^2 \).